

Exercice 1 :

a) dU différentielle totale exacte impose :

$$\left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial(l-P)}{\partial T}\right)_V \quad (1)$$

dS différentielle totale exacte impose :

$$\left(\frac{\partial(\frac{C_V}{T})}{\partial V}\right)_T = \left(\frac{\partial(\frac{l}{T})}{\partial T}\right)_V \quad (2)$$

$$(2) \Rightarrow \frac{1}{T} \left(\frac{\partial C_V}{\partial V}\right)_T = \frac{1}{T} \left(\frac{\partial l}{\partial T}\right)_V - \frac{l}{T^2}$$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial l}{\partial T}\right)_V - \frac{l}{T}$$

$$(1) \Rightarrow \left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial l}{\partial T}\right)_V - \left(\frac{\partial P}{\partial T}\right)_V$$

$$\frac{l}{T} = \left(\frac{\partial P}{\partial T}\right)_V \Rightarrow l = T \left(\frac{\partial P}{\partial T}\right)_V$$

$$2) \text{ a) } \left(\frac{\partial P}{\partial T}\right)_V = \frac{nR}{V} \Rightarrow l = \frac{nRT}{V} = \frac{PV}{V}$$

On trouve : $l = P$

On sait que : $\left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial(l-P)}{\partial T}\right)_V$

$\left(\frac{\partial C_V}{\partial V}\right)_T = 0 \Rightarrow C_V$ ne dépend pas de V

b) Comme $l = P$, alors : $dU = C_V dT$

C_V indépendant de $T \Rightarrow U = C_V T + Cste$

$$dS = \frac{C_V}{T} dT + \frac{P}{T} dV \Rightarrow dS = \frac{C_V}{T} dT + \frac{nR}{V} dV$$

$$S = C_V \ln T + nR \ln V + Cste$$

$$4) Q_A^B = U_B - U_A \Rightarrow Q_A^B = nC_V (T_B - T_A)$$

$$Q_C^A = H_A - H_C \Rightarrow Q_C^A = nC_P (T_A - T_C)$$

Détente adiabatique $\Rightarrow Q_B^C = 0$

5) Le cycle est décrit dans le sens des aiguilles d'une montre, alors : $W_{\text{cycle}} < 0$

Le rendement ρ vaut : $\rho = \frac{-W_{\text{cycle}}}{Q_{\text{récu}}} = \frac{-W_{\text{cycle}}}{Q_{AB}}$

$$6) \text{ CA isobare } \Rightarrow \frac{V}{T} = cste \Rightarrow \frac{T_C}{T_A} = \frac{V_C}{V_A}$$

$$\text{AB isochore } \Rightarrow \frac{P}{T} = cste \Rightarrow \frac{T_B}{T_A} = \frac{P_B}{P_A}$$

$$7) \text{ BC adiabatique et réversible } \Rightarrow PV^\gamma = cste$$

$$\frac{P_C}{P_B} = \frac{P_A}{P_B} = \left(\frac{V_B}{V_C}\right)^\gamma \Rightarrow \frac{1}{\alpha} = \left(\frac{V_A}{V_C}\right)^\gamma \Rightarrow \frac{V_C}{V_A} = \alpha^{\frac{1}{\gamma}}$$

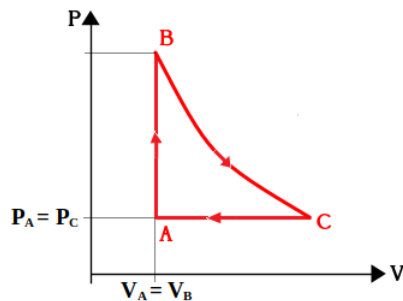
$$8) \rho = \frac{-W_{\text{cycle}}}{Q_A^B} = \frac{Q_A^B + Q_C^A}{Q_A^B} \text{ car } W_{\text{cycle}} + Q_{\text{cycle}} = 0$$

$$\rho = 1 + \frac{nC_P(T_A - T_C)}{nC_V(T_B - T_A)} \Rightarrow \rho = 1 - \frac{\gamma(T_C - T_A)}{T_B - T_A}$$

$$9) \frac{\frac{T_C}{T_A} - 1}{\frac{T_B}{T_A} - 1} = \frac{\frac{V_C}{V_A} - 1}{\frac{P_B}{P_A} - 1} = \frac{\alpha^{\frac{1}{\gamma}} - 1}{\alpha - 1} \Rightarrow \rho = 1 - \gamma \frac{\alpha^{\frac{1}{\gamma}} - 1}{\alpha - 1}$$

Exercice 2 :

1. Le diagramme de Clapeyron correspondant est :



2. Nous savons que :

$$\int dU = W + Q = - \int P dV + Q$$

A volume constant, $dV = 0 \Rightarrow \Delta U = Q$

3) Nous savons que :

$$\Delta H = \Delta U + V \Delta P + P \Delta V$$

$$\Delta H = Q - P \Delta V + V \Delta P + P \Delta V$$

$$P = cste \Rightarrow \Delta P = 0 \Rightarrow \Delta H = Q$$