

Exercice 1 :

1. On a :

$$\Delta S = nR \ln \left(\frac{P_1}{P_2} \right) \Rightarrow \Delta s = \frac{\Delta S}{m}$$

$$\Delta s = \frac{R}{M_{\text{air}}} \ln \left(\frac{P_1}{P_2} \right) \Rightarrow \Delta s = -0,462 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

2.

2.1 On a :

$$W_{12} = -nRT_1 \ln \left(\frac{V_2}{V_1} \right) = -nRT_1 \ln \left(\frac{P_1}{P_2} \right)$$

$$w_{12} = \frac{W_{12}}{m} \Rightarrow w_{12} = \frac{R}{M} T_1 \ln \left(\frac{P_1}{P_2} \right)$$

$$w_{12} = 133 \text{ kJ} \cdot \text{kg}^{-1}$$

$$Q_{12} = -W_{12} \Rightarrow q_{12} = -w_{12}$$

$$q_{12} = -133 \text{ kJ} \cdot \text{kg}^{-1}$$

$$\text{Comme } P_2 > P_1 \Rightarrow V_2 < V_1 \Rightarrow dV < 0$$

$$dW = -PdV \Rightarrow w_{12} > 0$$

2.2 Transformation réversible $\Rightarrow$: $s_c =$

$$0 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

$$s_e = \Delta s \Rightarrow s_e = -0,462 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

3.

$$3.1 \quad w'_{12} = -\frac{P_2(V_2-V_1)}{m} = -\frac{P_2 V_1}{m} \left(\frac{V_2}{V_1} - 1 \right)$$

$$w'_{12} = -\frac{R}{M} \frac{P_2}{P_1} T_1 \left(\frac{P_1}{P_2} - 1 \right)$$

$$\text{On trouve : } w'_{12} = 303,3 \text{ kJ} \cdot \text{kg}^{-1}$$

$$q'_{12} = -w'_{12} = -303,3 \text{ kJ} \cdot \text{kg}^{-1}$$

$$3.2 \quad s'_e = \frac{q'_{12}}{T_1} \Rightarrow s'_e = -1,147 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

$$s'_e < 0 \text{ car perte de chaleur}$$

$$s'_c = \Delta s - s'_e \Rightarrow s'_c = 0,685 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

$$s'_c > 0 \text{ car une entropie est créée.}$$

$$C'(P) = 0 \Rightarrow C(P) = C_{ste}$$

$$\text{D'où : } \mathbf{V}(T, P) = \frac{nRT}{P} + \text{constante}$$

3

$$3.a \quad dU = \frac{\partial U}{\partial T} dT + \frac{\partial U}{\partial V} dV \Rightarrow \begin{cases} \frac{\partial U}{\partial T} = c_V \\ \frac{\partial U}{\partial V} = l - P \end{cases}$$

$$\begin{cases} \frac{\partial^2 U}{\partial T \partial V} = \left(\frac{\partial c_V}{\partial V} \right)_T \\ \frac{\partial^2 U}{\partial V \partial T} = \left(\frac{\partial l}{\partial T} \right)_V - \left(\frac{\partial P}{\partial T} \right)_V \end{cases}$$

$$\text{On obtient : } \left(\frac{\partial c_V}{\partial V} \right)_T = \left(\frac{\partial l}{\partial T} \right)_V - \left(\frac{\partial P}{\partial T} \right)_V$$

$$dS = \frac{\partial S}{\partial T} dT + \frac{\partial S}{\partial V} dV \Rightarrow \begin{cases} \frac{\partial S}{\partial T} = \frac{c_V}{T} \\ \frac{\partial S}{\partial V} = \frac{l}{T} \end{cases}$$

$$\begin{cases} \frac{\partial^2 S}{\partial T \partial V} = \frac{1}{T} \left(\frac{\partial c_V}{\partial V} \right)_T \\ \frac{\partial^2 S}{\partial V \partial T} = \frac{1}{T} \left(\frac{\partial l}{\partial T} \right)_V - \frac{l}{T^2} \end{cases}$$

$$\frac{1}{T} \left(\frac{\partial c_V}{\partial V} \right)_T = \frac{1}{T} \left(\frac{\partial l}{\partial T} \right)_V - \frac{l}{T^2}$$

$$\text{On trouve : } \left(\frac{\partial c_V}{\partial V} \right)_T = \left(\frac{\partial l}{\partial T} \right)_V - \frac{l}{T}$$

$$3.b \quad \left(\frac{\partial c_V}{\partial V} \right)_T = \left(\frac{\partial l}{\partial T} \right)_V - \frac{l}{T} = \left(\frac{\partial l}{\partial T} \right)_V - \left(\frac{\partial P}{\partial T} \right)_V$$

$$\frac{l}{T} = \left(\frac{\partial P}{\partial T} \right)_V \Rightarrow l = T \left(\frac{\partial P}{\partial T} \right)_V$$

$$\left(\frac{\partial c_V}{\partial V} \right)_T = \left(\frac{\partial}{\partial T} \left(T \left(\frac{\partial P}{\partial T} \right)_V \right) \right)_V - \left(\frac{\partial P}{\partial T} \right)_V$$

$$\left(\frac{\partial c_V}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V + T \left(\frac{\partial^2 P}{\partial T^2} \right)_V - \left(\frac{\partial P}{\partial T} \right)_V$$

$$\text{On en déduit : } \left(\frac{\partial c_V}{\partial V} \right)_T = T \left(\frac{\partial^2 P}{\partial T^2} \right)_V$$

Exercice 2:

1. On a :

$$dV = \frac{\partial V}{\partial T} dT + \frac{\partial V}{\partial P} dP$$

$$2. \quad \begin{cases} dV = \frac{\partial V}{\partial T} dT + \frac{\partial V}{\partial P} dP \\ dV = \frac{nR}{P} dT - \frac{nRT}{P^2} dP \end{cases} \Rightarrow \begin{cases} \frac{\partial V}{\partial T} = \frac{nR}{P} \\ \frac{\partial V}{\partial P} = -\frac{nRT}{P^2} \end{cases}$$

$$\frac{\partial V}{\partial T} = \frac{nR}{P} \Rightarrow V(T, P) = \frac{nR}{P} T + C(P)$$

$$\frac{\partial V}{\partial P} = -\frac{nRT}{P^2} \Rightarrow V(T, P) = \frac{nR}{P} T + C(P)$$